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1. Primer Item

$$P(x) = x^5 - 6x^3 - 8x^2 - 3x$$

$\boxed{x=-1} \Rightarrow \boxed{x+1}=0$

$$\boxed{f'(x) = 5x^4 - 18x^2 - 16x - 3}$$

$$\begin{aligned} f(-1) &= 5(-1)^4 - 18(-1)^2 - 16(-1) - 3 \\ &= 5(1) - 18(1) + 16 - 3 \\ &= 5 - 18 + 16 - 3 \\ &= 0_{//} \end{aligned}$$

$$\boxed{f''(x) = 20x^3 - 36x - 16}$$

$$\begin{aligned} f(-1) &= 20(-1)^3 - 36(-1) - 16 \\ &= 20(-1) + 36 - 16 \\ &= -20 + 36 - 16 \\ &= 0_{//} \end{aligned}$$

$$\boxed{f'''(x) = 60x^2 - 36}$$

$$\begin{aligned} f(-1) &= 60(-1)^2 - 36 \\ &= 60(1) - 36 \\ &= 60 - 36 \\ &= 24_{//} \end{aligned}$$

$$(x+1)^3$$

2

$$\frac{x^5 - 6x^3 - 8x^2 - 3x}{x^3 + 3x^2 + 3x + 1} = P(x)$$

$$\cancel{x^5} - 6x^3 - 8x^2 - 3x \div x^3 + 3x^2 + 3x + 1 = x^2 - 3x$$

$$-(\cancel{x^5} + 3x^4 + 3x^3 + x^2)$$

$$-3x^4 - 9x^3 - 9x^2 - 3x$$

$$\underline{(+3x^4 + 9x^3 + 9x^2 + 3x)}$$

$$= 0$$

$$P(x) = x^2 - 3x$$

$$x(x-3) < \begin{array}{l} \boxed{x=0} \\ \boxed{x=3} \end{array}$$

3

$$f(x) = x^5 - 6x^3 - 8x^2 - 3x = x(x+1)^3(x-3)$$

Segundo

2. Item

a) $\displaystyle\sum_{n=1}^{\infty} \frac{2n^2}{(3n-2)(3n+1)} = a_n$

$$\lim_{n\to\infty} \frac{2n^2}{9n^2-3n-2} = \frac{2}{9} \cdot \lim_{n\to\infty} \overbrace{\frac{n^2}{n^2-\frac{1}{3}n-\frac{2}{9}}}^{\to 1} = \frac{2}{9}\cdot 1 = \frac{2}{9} = L$$

∴ Diverge.

b) $\displaystyle\sum_{n=1}^{\infty} \frac{(n+4)!}{2^n \cdot n^2 (n+1)!} = a_n$

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \frac{(n+5)!}{2^{n+1}(n+1)^2(n+2)!} \cdot \frac{2^n \cdot n^2 (n+1)!}{(n+4)!} = \frac{n^2(n+5)(n+1)}{2(n+1)^2(n+2)(n+4)} =$$

$$\lim_{n\to\infty} \left|\frac{n^2(n+5)(n+1)}{2(n+1)^2(n+2)(n+4)}\right| = \frac{1}{2} \lim_{n\to\infty} \overbrace{\left|\frac{n^2(n+5)(n+1)}{(n+1)^2(n+2)(n+4)}\right|}^{\to 1} = L = \frac{1}{2}\cdot 1 = \frac{1}{2} < 1$$

∴ Converge.

3. Tercer Item

a) $C_{n+1} = \dfrac{(-1)^{n+1} (x-5)^{n+1}}{2^{n+1} (3n+5)}$

$L = \lim\limits_{n\to\infty} \left| \dfrac{C_{n+1}}{C_n} \right| = \dfrac{\cancel{(-1)^{n+1}} \cancel{(x-5)^{n+1}}}{\cancel{2^{n+1}} (3n+5)} \cdot \dfrac{\cancel{2^n}(3n+2)}{\cancel{(-1)^n} \cancel{(x-5)^n}} =$

$\lim\limits_{n\to\infty} \left| \dfrac{(-1)(x-5)(3n+2)}{2(3n+5)} \right| = \left| \dfrac{x-5}{2} \right| \lim\limits_{n\to\infty} \left| \dfrac{3n+2}{3n+5} \right| \to 1$

$L = \dfrac{|x+5|}{2} < 1 \qquad |x+5| < 2 \Rightarrow -2 < x+5 < 2$

$3 < x < 7$

$\therefore$ CA $x \in I(3,7)$

caso 1 $\boxed{x=3}$

$(-1)^n (3-5)^n = \dfrac{(-1)^n (-2)^n}{2^n (3n+2)} = \dfrac{(-1)^n \cdot (-1)^n \cdot 2^n}{2^n (3n+2)} = \dfrac{1^{2n}}{3n+2} = a_n$

$\sum\limits_{1}^{\infty} \dfrac{1}{3n+2} = a_n \Rightarrow b_n = \dfrac{1}{3n} \Rightarrow \sum\limits_{1}^{\infty} \dfrac{1}{3n} \Rightarrow \dfrac{1}{3} \sum\limits_{1}^{\infty} \dfrac{1}{n} \Rightarrow p = 1$

$\therefore$ Diverge

caso 2 = $\boxed{x=7}$

$$\sum \frac{(-1)^n 2^n}{2^n (3n+2)} = \sum \left| \frac{(-1)^n}{(3n+2)} \right| = \frac{1}{3n+2}$$

Leibnitz

1) verificar que $a_n > 0$

$$\frac{1}{3n+2} > 0 \checkmark$$

2) verificar que $f'(n) < 0$

$$(3n+2)^{-1} = 3 \cdot -(3n+2)^{-2}$$

$$= \frac{-3}{(3n+2)^2} < 0 \checkmark$$

3) verificar $\lim\limits_{n \to \infty} a_n = 0$

$$\lim_{n \to \infty} \frac{1}{(3n+2)} = 0 \checkmark$$

$\therefore$ Por Leibnitz CA = C