

Ensayo segunda prueba
Algebra II

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RUT = 19.300.714-7

Primer item

$$P(x) = x^5 - 6x^3 - 8x^2 - 3x$$

$$f'(x) = 5x^4 - 18x^2 - 16x - 3$$

$$f(1) = 5(-1)^4 - 18(-1)^2 - 16(-1) - 3$$

$$= 5(1) - 18(1) + 16 - 3$$

$$= 5 - 18 - 16 - 3$$

$$= 0 //$$

$$f''(x) = 20x^3 - 36x - 16$$

$$f''(-1) = 20(-1)^3 - 36(-1) - 16$$

$$= 20(-1) + 36 - 16$$

$$= -20 + 36 - 16$$

$$= 0 //$$

$$f'''(x) = 60x^2 - 36$$

$$f'''(-1) = 60(-1)^2 - 36$$

$$= 60 - 36$$

$$= 24 //$$

$$(x+1)^3$$

$$x = -1 \Rightarrow x + 1 = 0$$

$$\textcircled{2} \frac{x^5 - 6x^3 - 8x^2 - 3x}{x^3 + 3x^2 + 3x + 1} = P(x)$$

$$x^5 - 6x^3 - 8x^2 - 3x \div x^3 + 3x^2 + 3x + 1 = x^2 - 3x$$

$$-(x^5 + 3x^4 + 3x^3 + x^2)$$

$$-3x^4 - 9x^3 - 9x^2 - 3x$$

$$3x^4 + 9x^3 + 9x^2 + 3x$$

$$= 0$$

$$P(x) = x^2 - 3x$$

$$x(x-3) \begin{cases} \rightarrow x=0 \\ \rightarrow x=3 \end{cases}$$

$$\textcircled{3} f(x) = x^5 - 6x^3 - 8x^2 - 3x$$

$$= x(x+1)^3(x-3)$$

Segundo Item.

$$a) \sum_{n=1}^{\infty} \frac{2n^2}{(3n-2)(3n+1)} = a_n$$

$$\lim_{n \rightarrow \infty} \frac{2n^2}{9n^2 - 3n - 2} = \frac{2}{9} \lim_{n \rightarrow \infty} \frac{n^2}{n^2 - \frac{1}{3}n - \frac{2}{9}} = \frac{2}{9} \cdot 1 = \frac{2}{9} = L$$

$\therefore$ Diverge

$$b) \sum_{n=1}^{\infty} \frac{(n+4)!}{2^n \cdot n^2 (n+1)!} = a_n$$

$$\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \frac{(n+5)!}{2^{n+1} (n+1)^2 (n+2)!} \cdot \frac{2^n \cdot n^2 (n+1)!}{(n+4)!}$$

$$= \frac{n^2 (n+5)(n+1)}{2(n+1)^2 (n+2)(n+4)}$$

$$\lim_{n \rightarrow \infty} \left| \frac{n^2 (n+5)(n+1)}{2(n+1)^2 (n+2)(n+4)} \right| = \frac{1}{2} \lim_{n \rightarrow \infty} \left| \frac{n^2 (n+5)(n+1)}{(n+1)^2 (n+2)(n+4)} \right| = L = \frac{1}{2} \cdot 1 = \frac{1}{2} < 1$$

$\therefore$ Converge

tercer Item

$$c) C_{n+1} = \frac{(-1)^{n+1} (x-5)^{n+1}}{2^{n+1} (3n+5)}$$

$$L = \lim_{n \rightarrow \infty} \left| \frac{C_{n+1}}{C_n} \right| = \frac{(-1)^{n+1} (x-5)^{n+1}}{2^{n+1} (3n+5)} \cdot \frac{2^n (3n+2)}{(-1)^n (x-5)^n} =$$

$$\lim_{n \rightarrow \infty} \left| \frac{(-1)(x-5)(3n+2)}{2(3n+5)} \right| = \left| \frac{x-5}{2} \right| \lim_{n \rightarrow \infty} \left| \frac{3n+2}{3n+5} \right|$$

$$L = \frac{|x+5|}{2} < 1 \quad |x+5| < 2 \Rightarrow -2 < x+5 < 2 \Rightarrow 3 < x < 7$$

$\therefore \textcircled{CA} x \in I(3,7)$

Caso 1 $|x=3|$

$$\frac{(-1)^n (3-5)^n}{2^n (3n+2)} = \frac{(-1)^n (-2)^n}{2^n (3n+2)} = \frac{(-1)^n \cdot (-1)^n \cdot 2^n}{2^n (3n+2)} = \frac{1^{2n}}{3n+2} = a_n$$

$$\sum_{n=1}^{\infty} \frac{1}{3n+2} = a_n \Rightarrow b_n = \frac{1}{3n} \Rightarrow \sum_{n=1}^{\infty} \frac{1}{3n} \Rightarrow \frac{1}{3} \sum_{n=1}^{\infty} \frac{1}{n} \Rightarrow p=1$$

$\therefore$ Diverge

Caso 2 $|x=7|$

$$\sum \frac{(-1)^n 2^n}{2^n (3n+2)} = \sum \frac{(-1)^n}{(3n+2)} = \frac{1}{3n+2}$$

Leibnitz

1) Verificar que $a_n > 0$

$$\frac{1}{3n+2} > 0$$

2) Verificar que $f'(n) < 0$

$$\begin{aligned} (3n+2)^{-1} &= 3 \cdot -(3n+2)^{-2} \\ &= \frac{-3}{(3n+2)^2} < 0 \end{aligned}$$

3) Verificar $\lim_{n \rightarrow \infty} a_n = 0$

$$\lim_{n \rightarrow \infty} \frac{1}{(3n+2)} = 0$$

$\therefore$ per Leibnitz $(C_A) = (C)$