Problema 1

$x^5 - 6x^3 - 8x^2 - 3x \qquad x = -1$

a) $P(x) = x^5 - 6x^3 - 8x^2 - 3x = 0$ Multiplicidad $(x+1)^3$

$P'(x) = 5x^4 - 18x^2 - 16x - 3$

$P(-1) = 5 - 18 + 16 - 3 = 0$

$P''(x) = 20x^3 - 36x - 16$

$P''(-1) = -20 + 36 - 16 = 0$

$P'''(x) = 60x^2 - 36$

$P'''(-1) = 60 - 36 = 24 \neq 0$

b) $\dfrac{x^5 - 6x^3 - 8x^2 - 3x}{(x+1)^3} =$

Problema 2

a) $\sum_{1}^{\infty} \frac{2n^2}{(3n-2)(3n+1)}$ $\quad a_n = \frac{2n^2}{9n^2-3n-2}$ $\quad \lim_{n\to\infty} \frac{2n^2}{9n^2-3n-2} = \lim_{n\to\infty} \frac{2\cancel{n^2}}{9\cancel{n^2}} = \frac{2}{9} \neq 0$

Diverge
Por criterio de la divergencia

b) $\sum_{1}^{\infty} \frac{(n+4)!}{2^n n^2 (n+1)!} = \frac{(n+4)\cancel{n!}}{2^n n^2 (n+1)\cancel{n!}} \Rightarrow a_n = \frac{n+4}{2^n n^2 (n+1)}$

$a_{n+1} = \frac{n+5}{2^{n+1}(n+1)^2(n+2)}$

$\frac{a_{n+1}}{a_n} = \frac{n+5}{2^{\cancel{n+1}}(n+1)^{\cancel{2}}(n+2)} \cdot \frac{\cancel{2^n} n^2 \cancel{(n+1)}}{n+4} = \frac{n^2(n+5)}{2(n+1)(n+2)(n+4)} = \frac{n^3+5n^2}{2n^3+14n^2+28n+16}$

$L < 1$

$\lim_{n\to\infty} \frac{n^3+5n^2}{2n^3+14n^2+28n+16} = \lim_{n\to\infty} \frac{\cancel{n^3}}{2\cancel{n^3}} = \frac{1}{2}$

Converge
Por criterio de la razon

Problema 3

$$\sum_{1}^{\infty} \frac{(-1)^n (x-5)^n}{2^n (3n+2)} = a_{n+1} = \frac{(-1)^{n+1} (x-5)^{n+1}}{2^{n+1} (3n+3)}$$

$$\frac{a_{n+1}}{a_n} = \frac{(-1)^{\cancel{n}+1} (x-5)^{\cancel{n}+1}}{2^{\cancel{n}+1} (3n+3)} \cdot \frac{\cancel{2^n} (3n+2)}{\cancel{(-1)^n} \cancel{(x-5)^n}}$$

$$\frac{a_{n+1}}{a_n} = \frac{(-1)(x-5)(3n+2)}{2(3n+3)}$$

$$\left|\frac{a_{n+1}}{a_n}\right| = \frac{|x-5|\,|3n+2|}{|2|\,|3n+3|} = \frac{|x-5|(3n+2)}{2(3n+3)}$$

$$L = \lim_{n\to\infty} \frac{|x-5|(3n+2)}{2(3n+3)} = \frac{|x-5|}{2} \lim_{n\to\infty} \frac{3n+2}{3n+3} = \frac{\cancel{3n}}{\cancel{3n}} = 1$$

$$L < 1 \rightarrow \frac{|x-5|}{2} < 1 \rightarrow |x-5| < 2$$

$-2 < x-5 < 2$

$3 < x < 7$

El intervalo de convergencia absoluta es $(3,7)$

Se evalua la serie en $x = 3$
$x = 7$

$$\sum_{1}^{\infty} \frac{(-1)^n (x-5)^n}{2^n (3n+2)} \underset{x=3}{=} \sum_{1}^{\infty} \frac{(-1)^n (-2)^n}{2^n (3n+2)} = \sum_{1}^{\infty} \frac{(-1)^n (-1)^n \cancel{(2)}^n}{\cancel{2^n} (3n+2)}$$

a_n

$$\sum_{1}^{\infty} \frac{(-1)^{2n}}{3n+2}$$ Converge a cero

$x = 7$

$$\sum^{\infty} \frac{(-1)^n (7-5)^n}{2^n (3n+2)} = \sum_{1}^{\infty} \frac{(-1)^n \cancel{(2)}^n}{\cancel{2^n} (3n+2)} = \sum^{\infty} \frac{(-1)^n}{3n+2}$$

b_n