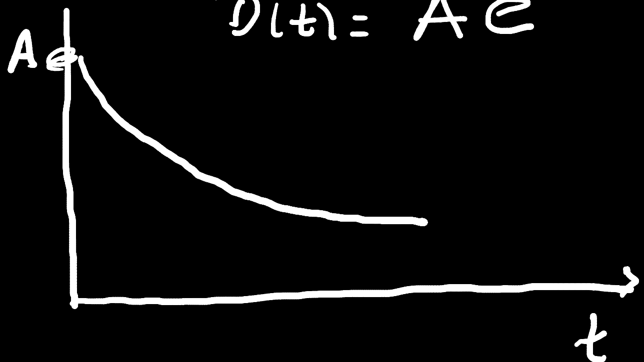
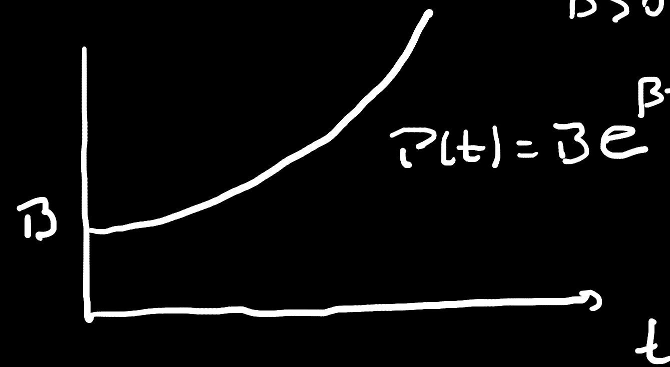
11/06/2025 MODELOS EXPONENCIALES

$D(t) = A e^{-\alpha \cdot t}$; $\alpha > 0$



$\beta > 0$

$P(t) = B e^{\beta t}$



$$y = k e^{-\alpha t} \quad /$$

$$\frac{y}{k} = e^{-\alpha t} \quad / \ln$$

$$\ln\left(\frac{y}{k}\right) = \ln\left(e^{-\alpha t}\right) = -\alpha t$$

$$\ln\left(\frac{y}{k}\right) = -\alpha t$$

Ejemplo

$$\boxed{N(t) = 2\, e^{-\alpha \cdot t}} \rightarrow \text{MODELO}$$

¿α? $\alpha > 0$

$N(4) = 0.3$ ✓ DATO

$$N(4) = 2 \cdot e^{-\alpha \cdot 4} = 0.3$$

$$2 \cdot e^{-\alpha \cdot 4} = 0.3$$

$$e^{-\alpha \cdot 4} = \frac{0.3}{2} = 0{,}15$$

Nota:

$\ln(a)$ es negativo si $0 < a < 1$

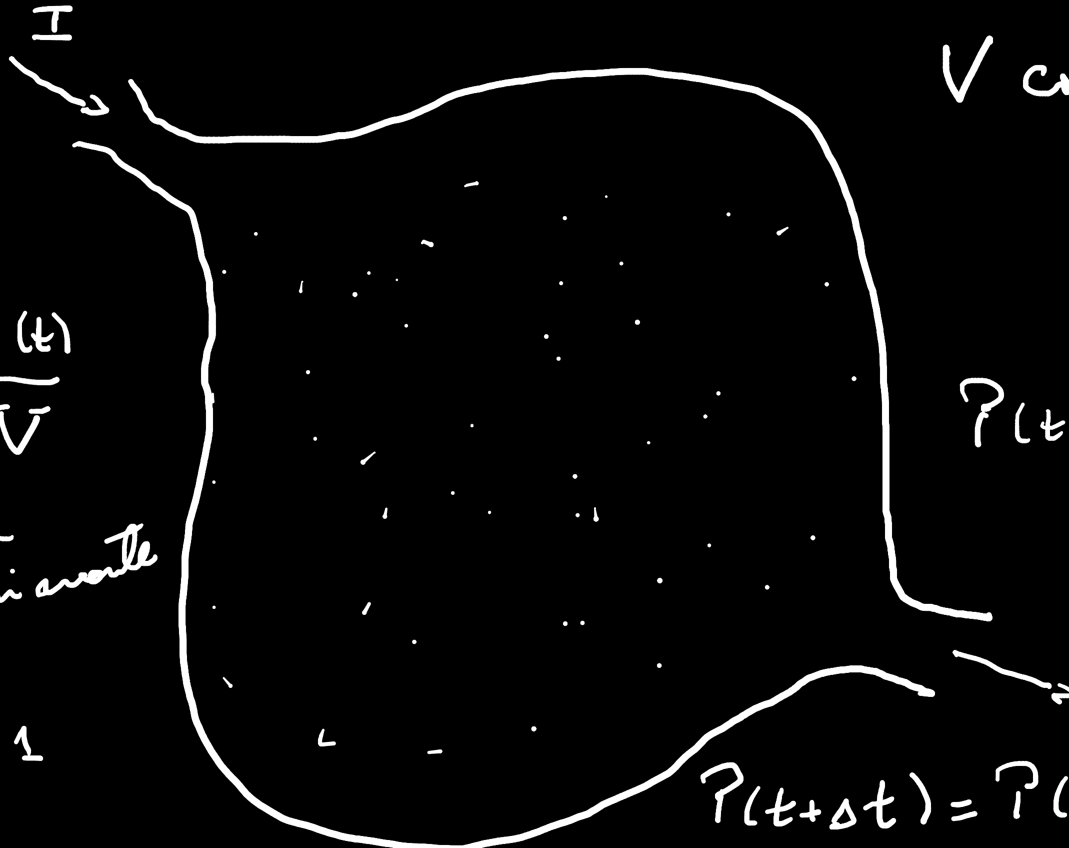
$$\therefore e^{-\alpha \cdot 4} = 0{,}15 \quad / \ln$$

$$-\alpha \cdot 4 = \ln 0.15$$

$$\alpha \cdot (-4) = -\alpha \cdot 4 = -1.89712$$

$$\therefore \alpha = \frac{-1.89712}{-4} = 0.47428 \checkmark$$

Modelo de contaminación de un lago.



V constante

$[V] = m^3$

$[I] = \dfrac{m^3}{seg.}$

$P(t)$: Volumen de contaminante

$P(0)$

I

$\rho(t) = \dfrac{P(t)}{V}$ → tasa de contaminante

$0 < \rho(t) < 1$

$$P(t+\Delta t) = P(t) - \underbrace{\rho(t) \cdot I \cdot \Delta t}_{} \quad / \; \frac{1}{V}$$

$$\frac{P(t+\Delta t)}{V} = \frac{P(t)}{V} - \rho(t) \frac{I}{V} \Delta t$$

$$\rho(t+\Delta t) = \rho(t) - \rho(t) \frac{I}{V} \Delta t$$

$$\rho(t) = \rho(0) \cdot e^{-\frac{I}{V} \cdot t}$$

−

$$\boxed{\rho(t) = \rho(0)\, e^{-\frac{I}{V}\cdot t}}$$

Tarea. Graficar este modelo de contaminación, $\rho(t)$, con los siguientes datos

$$V = 10^{6}\, m^{3}$$

$$I = 1\, \frac{m^{3}}{seg} \quad ; \quad \rho(0) = 0.1$$

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$$\frac{I}{V} = \frac{1}{10^{6}} = 10^{-6}$$

$$\left[\frac{I}{V}\right] = \left[\frac{1}{seg}\right]$$

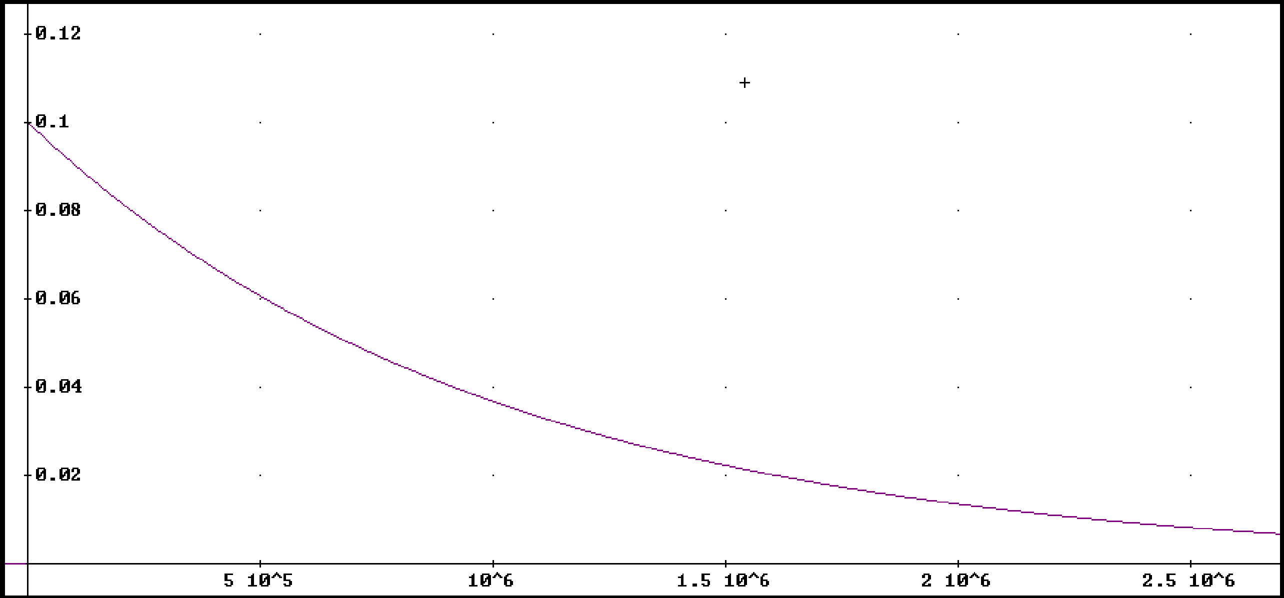
$\therefore$

$$\rho(t) = 0.1\, e^{-10^{-6}\cdot t}$$

$$\rho(t) = 0.1\, e^{-10^{-6}\cdot t}$$

Nota $\quad 1$ día $= 86.400$ seg

$$1 \text{ día} = 24 \cdot h = 24 \cdot 60 \text{ min} = \underbrace{24 \cdot 60 \cdot 60 \text{ seg}}_{86.400 \text{ seg}}.$$



$2 \cdot 10^6$ seg = ¿días? $\doteq$ 23 días.

1 día = 86.400 seg.

? días = $2 \cdot 10^6$ seg.

$$\frac{2 \cdot 10^6}{86.400} = \frac{2000000}{86400} \simeq 23 \text{ días.}$$

¿En cuántos días la contaminación inicial se reduce a la mitad?

$\rho(t) = 0.1 \cdot e^{-10^{-6} \cdot t}$ ¿t_0 tal que $\rho(t_0) = 0.05$?

$$\rho(t_0) = 0.05 = 0.1\, e^{-10^{-6} \cdot t_0}$$

$$0.05 = 0.1\, e^{-10^{-6} \cdot t_0}$$

$$\frac{0.05}{0.1} = e^{-10^{-6} \cdot t_0}$$

$$\frac{1}{2} = e^{-10^{-6} \cdot t_0} \quad / \ln$$

$$\ln(1/2) = -10^{-6} \cdot t_0$$

$$t_0 = \frac{\ln(1/2)}{-10^{-6}} = \frac{-0.693}{-10^{-6}} = 0.693 \times 10^{6}$$

$$t_0 = 693.000 \text{ seg} \doteq 8 \text{ días} \; //$$