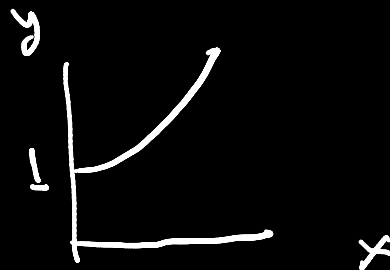


09/09/2025

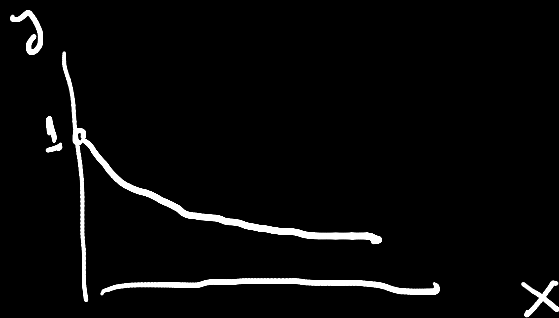
El modelo exponencial

$$e^{0.2x}$$



$$x \geq 0$$

$$e^{-0.3x}$$



$$e^{xx} > 0$$

$$e^x = y \iff \ln y = x$$

$$e' = e \quad \ln e = 1$$

$\ln \equiv$ logaritmo natural

¿Pone qué sirve $\ln$?

Sea la ecuación

$$e^{2x} = 3$$

¿ x ?

Aplicamos $\ln$

$$\ln(e^{2x}) = \ln(3)$$



$$2x = \ln(3)$$

$$x = \frac{\ln(3)}{2} = 0.549$$

Otro ejemplo

$$3 e^{0.5 t} = 45$$

c't?

$$e^{0.5 t} = 15 \quad / \ln$$

$$\ln(e^{0.5 t}) = \ln(15)$$

$$0.5 \cdot t = \ln(15)$$

$$t = \frac{\ln(15)}{0.5}$$

$$t = 2 \ln(15)$$

$$t = 5.416$$

A menudo nos encontramos con ecuaciones
del tipo

$$A = k e^{\alpha \cdot t}$$

, A, k y α
constantes
o t?

$$\frac{A}{k} = e^{\alpha t} \quad / \ln$$

$$\ln \left(\frac{A}{k} \right) = \ln e^{\alpha t}$$

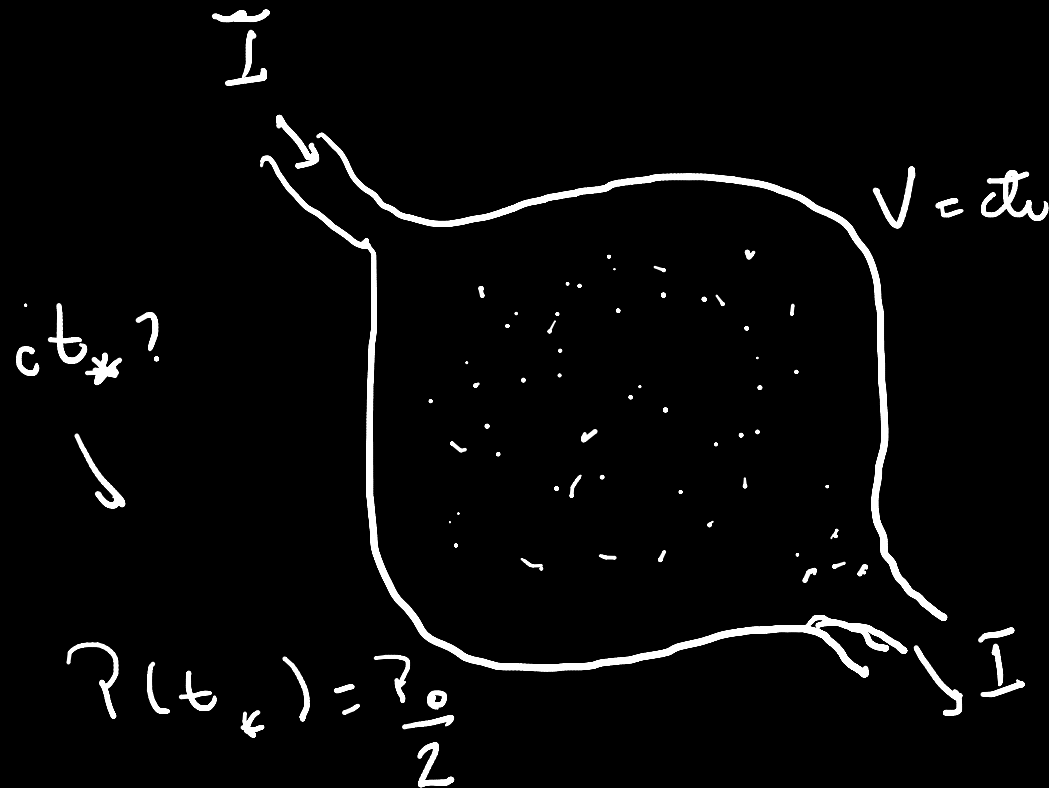
$$\ln \left(\frac{A}{k} \right) = \alpha \cdot t$$

$$t = \frac{1}{\alpha} \ln \left(\frac{A}{k} \right)$$

A y k tienen el mismo signo

Problema. Se desea minimizar de un
 lago sobre la fórmula

$$I(t) = I_0 e^{-0.1 \cdot t}$$



$$t=0$$

$$P(0) = P_0$$

Valor de
 solución inicial

$$I(t) = I_0 e^{-0.1 \cdot t}$$

$$[t] = \text{días}$$

¿ t_* tal que $I(t_*) = \frac{I_0}{2}$?

$$I(t_*) = I_0 e^{-0.1 \cdot t_*} = \frac{I_0}{2}$$

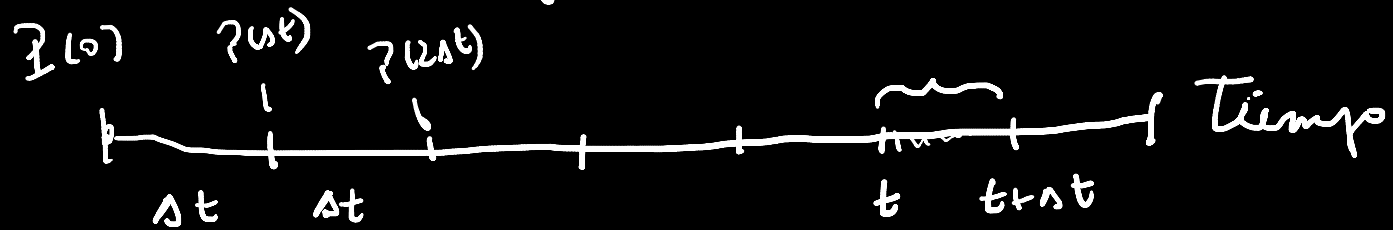
$$I_0 e^{-0.1 \cdot t_*} = \frac{I_0}{2}$$

$$e^{-0.1 \cdot t_*} = \frac{1}{2} \quad / \ln$$

$$-0.1 \cdot t_* = \ln(0.5)$$

$$t_* = \frac{\ln(0.5)}{-0.1} = -10 \ln(0.5) = 6.9 \text{ [días]}$$

¿Dónde aparece la función exponencial?



$P(t)$: Probabilidad en el tiempo t

$$P(t+\Delta t) = P(t) + \alpha \cdot P(t) \cdot \Delta t$$

$$\underbrace{P(t+\Delta t) - P(t)} = \alpha P(t) \cdot \Delta t$$

$$\frac{P(t+\Delta t) - P(t)}{\Delta t} = \alpha P(t)$$

$$\Delta t \rightarrow 0$$

$$\boxed{P(t) = P(0) \cdot e^{\alpha \cdot t}}$$