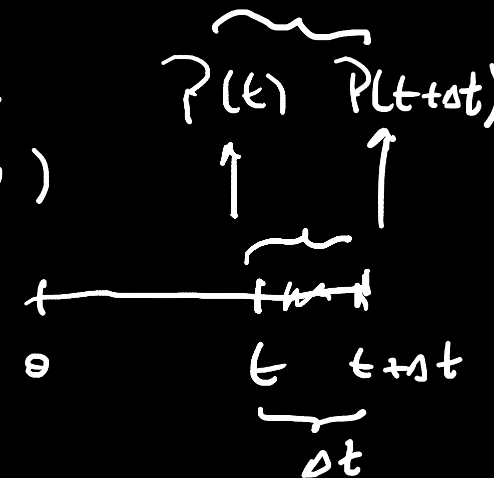
22/09/2025 Modelo exponencial

¿Cómo emerge $Ae^{\alpha t}$?

$P(t)$: Población en el tiempo t

$\underbrace{P(0)}_{P_0}$: Población inicial $(t=0)$



α : tasa de crecimiento

$$(*) \quad P(t+\Delta t) = P(t) + \alpha P(t)\cdot \Delta t$$

$$P(t+\Delta t) - P(t) = \alpha P(t)\cdot \Delta t$$

$$\frac{P(t+\Delta t) - P(t)}{\Delta t} = \alpha P(t)$$

$$\underbrace{\frac{P(t+\Delta t)-P(t)}{\Delta t}}_{\Delta t \to 0} = \alpha P(t)$$

$$\boxed{P'(t) = \alpha P(t)} \qquad ¿P(t)?$$

E.D.L.

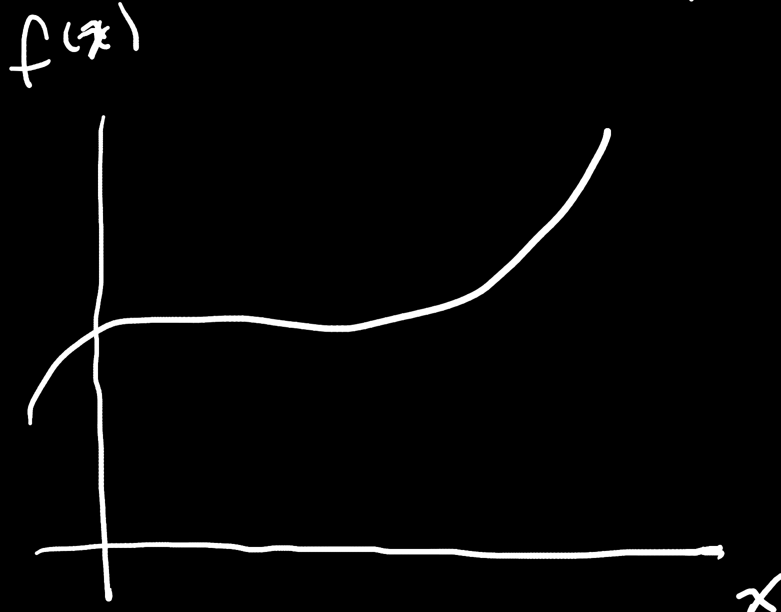
Solución: $P(t) = P_0 e^{\alpha \cdot t}$ ✓

E.D.L: Ecuación diferencial lineal

Vamos al EXCEL

DERIVADAS

Sea $f(x)$ una función real con variable
con x real



Ejemplo

$f(x) = 3x + 2$

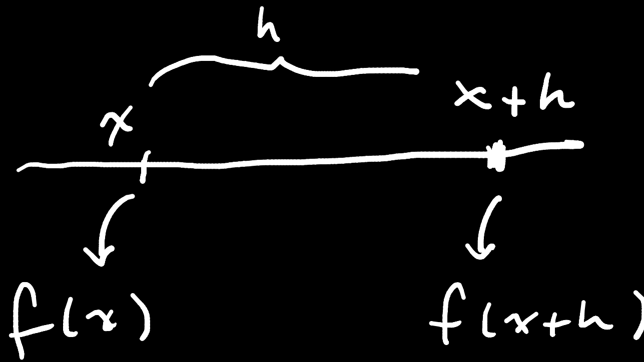
$f(x) = 5x^2 + 2x - 1$

$f(x) = e^{-3x}$

$f(x) = \ln(x)$

$f(x) = 3x^4 - 2x^3 + 5$

$f(x)$



$$\frac{f(x+h) - f(x)}{h} = f'(x)$$

$h \to 0$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \underbrace{f'(x)}_{\frac{d f(x)}{dx}}$$

Ejemplo : $f(x) = 3x^2 + x - 1$

$$f(x+h) = 3(x+h)^2 + x + h - 1$$
$$= 3(x^2 + 2xh + h^2) + x + h - 1$$
$$= 3x^2 + 6xh + 3h^2 + x + h - 1$$

$$f(x+h) - f(x) = \cancel{3x^2} + 6xh + 3h^2 + \cancel{x} + h \cancel{-1} - \cancel{3x^2} - \cancel{x} + \cancel{1}$$
$$f(x+h) - f(x) = 6xh + 3h^2 + h$$
$$\frac{f(x+h) - f(x)}{h} = \frac{6xh + 3h^2 + h}{h} = \frac{h(6x + 1 + 3h)}{h} =$$
$$= 6x + 1 + 3h \longrightarrow 6x + 1.$$

$h \to 0$ $\therefore$ $f'(x) = 6x + 1$

Otro ejemplo : $g(x) = 5x + 20$

$$\lim_{h \to 0} \frac{g(x+h) - g(x)}{h} = g'(x) \quad ?$$

$g(x+h) = 5(x+h) + 20 = 5x + 5h + 20$

$g(x+h) - g(x) = 5x + 5h + 20 - 5x - 20$

$g(x+h) - g(x) = 5h \quad \therefore \quad \frac{g(x+h) - g(x)}{h} = \frac{5h}{h} = 5$

$\therefore \quad g'(x) = 5 \quad //$

Otro ejemplo. $f(x) = 3x^2$

$$f(x+h) = 3(x+h)^2 = 3(x^2 + 2xh + h^2)$$

$$f(x+h) = 3x^2 + 6xh + 3h^2$$

$$f(x+h) - f(x) = 6xh + 3h^2 = h(6x+3h)$$

$$\frac{f(x+h) - f(x)}{h} = \frac{h(6x+3h)}{h} = 6x + 3h$$

$$\lim_{h \to 0} 6x + 3h = 6x$$

$$\therefore \quad f'(x) = 6x$$

$$f(x) = 5x^3 + 2x^2 + 4x + 20$$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = f'(x)$$

$$f'(x) = 15x^2 + 4x + 4$$

$$g(x) = 2x^7 + 5x^3 + 25x + 300$$

$$g'(x) = 14x^6 + 15x^2 + 25$$

$$h(x) = 2x^4 + 5x^2 + 300x + 10$$

$$h'(x) = 8x^3 + 10x + 300$$

$$f(x) = a\, x^{n}$$

a y n ctes.

$$f'(x) = a \cdot n \cdot x^{n-1}$$

$$g(x) = 6x^{3} \quad ; \quad g'(x) = 18x^{2}$$

$$\longrightarrow \quad f(x) = 3x^{2}$$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = 6x$$