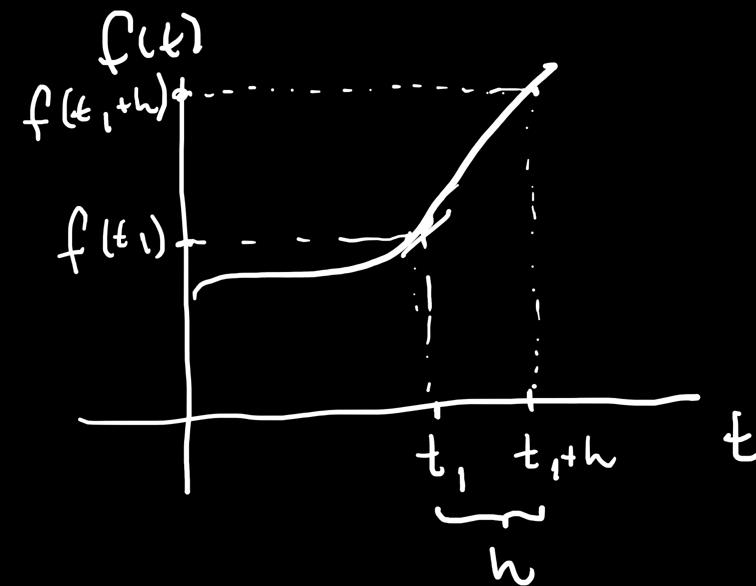
Biología Marina. 25/09/2023

DERIVADA de una función

Sea la función $f(t)$



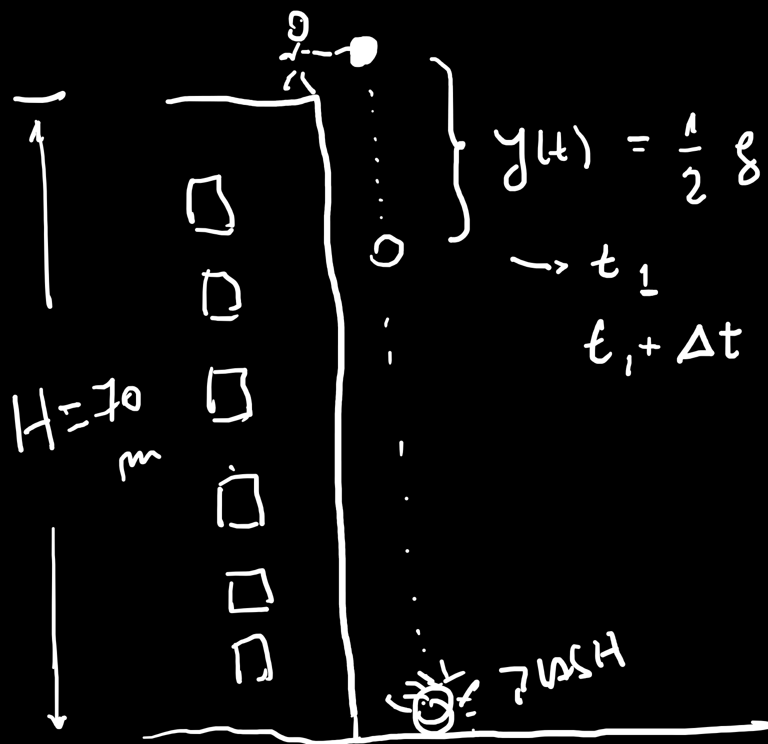
$$\lim_{h\to 0} \frac{f(t_1+h)-f(t_1)}{h} = f'(t_1)$$

$h \to 0$

LÍMITE

Veamos un ejemplo real:

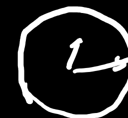
La ley de la caída libre de los cuerpos



$$y(t) = \frac{1}{2} g t^2$$

$\to t_1$

$t_1 + \Delta t$



$$g \doteq 9.8 \frac{m}{seg^2} \simeq 10 \frac{m}{sg^2}$$

$$\doteq 9.8 \frac{m/seg}{seg}$$

$$y(2) \doteq 20\ m.$$

$$y(3) = 45\ m$$

$$y(t_1) = \frac{1}{2} g t_1^2$$

$$y(t_1+\Delta t) = \frac{1}{2} \cdot g \cdot (t_1+\Delta t)^2$$

$$y(t_1+\Delta t) - y(t_1) = \frac{1}{2} g \cdot (t+\Delta t)^2 - \frac{1}{2} g t^2$$

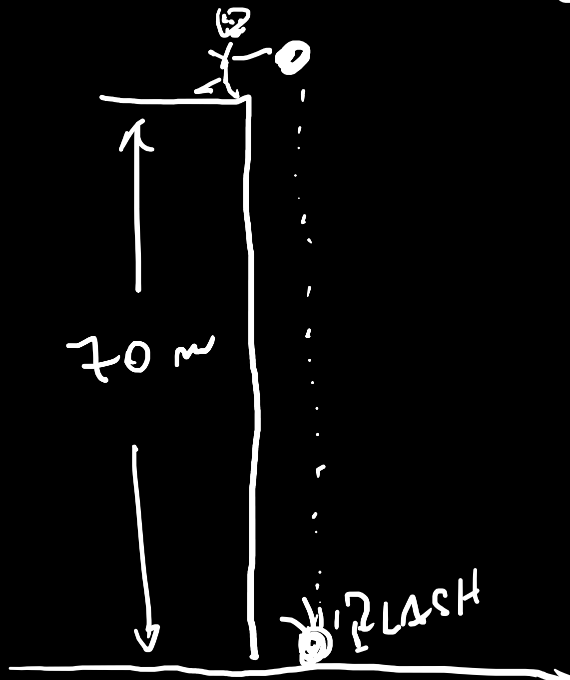
$$\therefore \quad = \frac{1}{2} g \left[t^2 + 2 t \cdot \Delta t + (\Delta t)^2 \right] - \frac{1}{2} g t^2$$

$$y(t_1+\Delta t) - y(t) = g t \Delta t + \frac{1}{2} g (\Delta t)^2$$

$$\frac{y(t_1+\Delta t) - y(t)}{\Delta t} = \frac{g t_1 \Delta t + \frac{1}{2} g (\Delta t)^2}{\Delta t} = g \cdot t_1 + \underbrace{\frac{1}{2} g \Delta t}_{0} \xrightarrow[\Delta t \to 0]{} g t$$

$$\therefore \quad v(t_1) = g \cdot t_1 \left[\frac{m}{seg} \right]$$

$$V(3) \doteq 30 \frac{m}{seg}$$



$$y(t) = \frac{1}{2} g t^2$$

$$v(t) = g \cdot t$$

¿velocidad de impacto?

¿Tiempo que tarda el tomate en recorrer 70 metros?

$t_i \doteq$ Tiempo de impacto

$$y(t_i) = 70$$

$$\frac{1}{2} g t_i^2 = 70$$

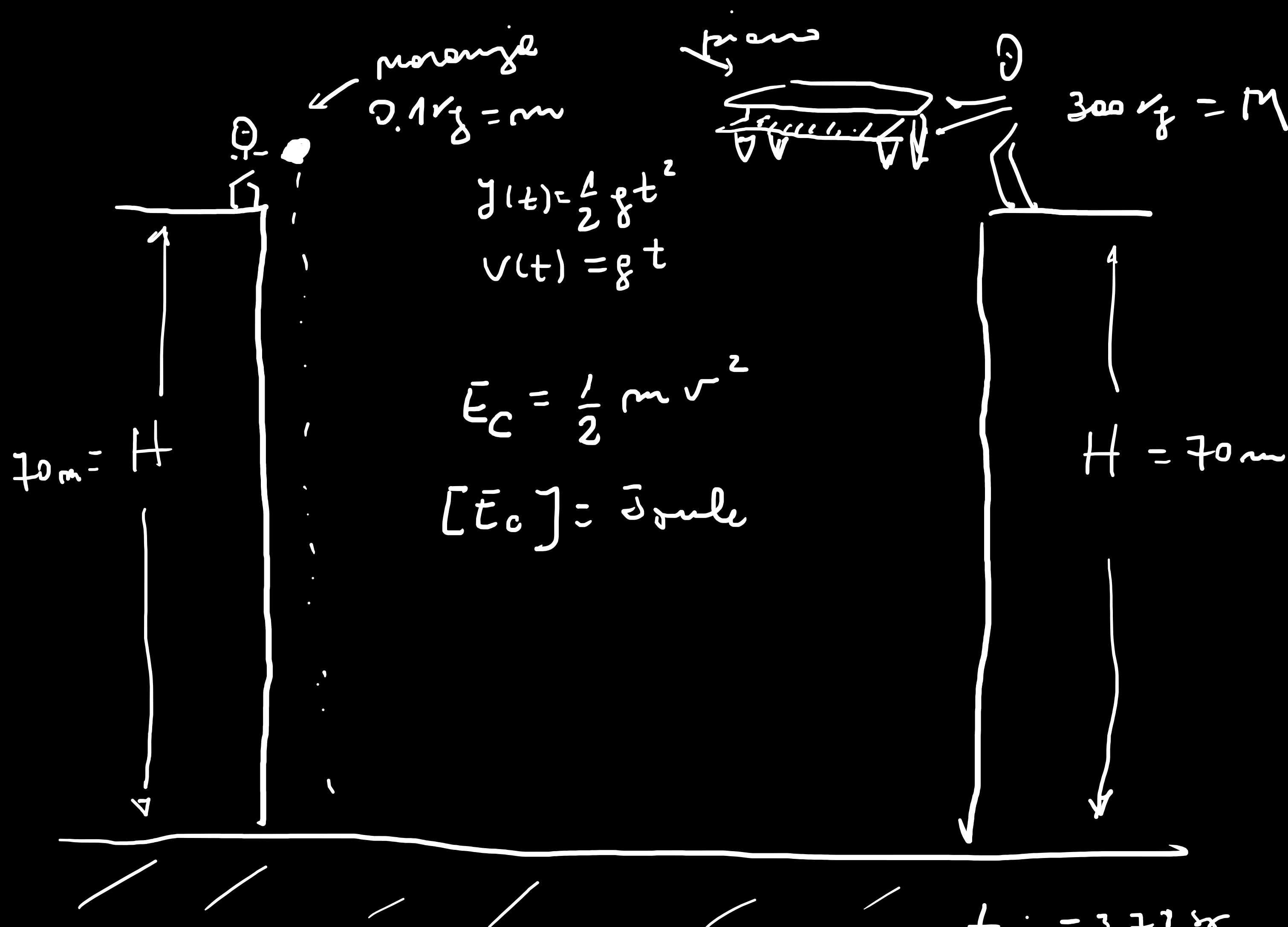
$$g t_i^2 = 140$$

$$t_i^2 = \frac{140}{g}$$

$$\therefore\ t_i = \sqrt{\frac{140}{g}} = \sqrt{\frac{140}{9.8}} = 3.78 \text{ seg.}$$

y entonces la velocidad de impacto es de

$$v(3.78) = g \cdot 3.78 \doteq 37.05 \frac{m}{seg}$$



$y(t) = \frac{1}{2} g t^2$

$v(t) = g t$

$E_c = \frac{1}{2} m v^2$

$[E_c] = \text{Joule}$

$t_i = 3.78$

$v_i = 37.05 \, \frac{m}{sg}$

$E_{cm} = 68.6 \, [J]$

$t_i = 3.78 \, sg$

$v_i = 37.05 \, \frac{m}{sg}$

$E_{cp} = 205905 \, [J]$

$$1 \text{ Joule} = \underbrace{1 \cdot \text{Newton}}_{\text{Fuerza}} \cdot \underbrace{1 \text{ metro}}_{\text{desplazamiento}}$$

$$1 \text{ Newton} = 1 \vec{N} = 1 \text{ kg} \cdot \frac{\text{m}}{\text{sg}^2}$$

$$\downarrow$$

$$\text{Fuerza} = \text{masa} \times \text{aceleración}$$

26/09/2023. Seguimos con derivadas

$$\lim_{\Delta t \to 0} \frac{f(t+\Delta t) - f(t)}{\Delta t} = f'(t)$$

Propiedades

1) $f(t) = cte$; $f'(t) = 0$

$f(t+\Delta t) = cte$ $\therefore$ $f(t+\Delta t) - f(t) = 0$

2) $f(t) = t^n$; $f'(t) = n t^{n-1}$

3) $[f(t) + g(t)]' = f'(t) + g'(t)$

$$f(t) = \operatorname{sen}(t)$$

$$\lim_{\Delta t \to 0} \frac{f(t+\Delta t) - f(t)}{\Delta t} =$$

$$= \lim_{\Delta t \to 0} \frac{\operatorname{sen}(t+\Delta t) - \operatorname{sen}(t)}{\Delta t} = \cos(t)$$

$$\therefore \; (\operatorname{sen}(t))' = \cos(t)$$

y además

$$(\cos t)' = -\operatorname{sen}(t)$$

Sea $g(x) = 3x^4 + 2x^3 - 5x^2 + x - 1$

$g'(x) = 12x^3 + 6x^2 - 10x + 1$

Nota $x = x^1$

$(x)' = 1 \cdot x^{1-1} = 1 \cdot x^0 = 1$

Ejemplo $\frac{d}{dx}(6x^4) = 24x^3$

Composición de funciones

1)
$$\begin{tikzcd} x \arrow[r] \arrow[rr, bend right] & x^2 \arrow[r] & \operatorname{sen}(x^2) \end{tikzcd}$$

$$x \longrightarrow \operatorname{sen}(x^2)$$

2)
$$\begin{tikzcd} x \arrow[r] \arrow[rr, bend right] & 3x^2+1 \arrow[r] & \sqrt{3x^2+1} \end{tikzcd}$$

$$x \longmapsto \sqrt{3x^2+1}$$

¿Cómo calculamos las derivadas?

$$x \xrightarrow{f} f(x) \xrightarrow{g} g(f(x))$$

$$x \longrightarrow g(f(x))$$

$$[g(f(x))]' = g'(f(x)) \cdot f'(x)$$

¿Cómo se hace?

Lo veremos en un ejemplo

$$[\operatorname{sen}(x^2)]' = \cos(x^2) \cdot 2x$$

$$\underbrace{[\operatorname{sen}(x^2)]'}_{\frac{d(\operatorname{sen} x^2)}{dx}} = 2x \cdot \cos(x^2)$$

$$\frac{d(\operatorname{sen} x^2)}{dx} = 2x \cos(x^2)$$

sco

$$f(x) = \sqrt{x} \quad ; \quad x \geq 0$$

$$\frac{d\sqrt{x}}{dx} = \frac{d}{dx}\left(x^{1/2}\right) = \frac{1}{2} x^{\frac{1}{2}-1}$$

$$= \frac{1}{2} x^{-1/2} = \frac{1}{2\ x^{1/2}} = \frac{1}{2\sqrt{x}}$$

$$\therefore \underbrace{\frac{d\sqrt{x}}{dx}}_{\left(\sqrt{x}\right)'} = \frac{1}{2\sqrt{x}}$$

$$\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}} \quad ; \quad x \geq 0$$

1) Sea $f(x) = e^x$

$$\frac{d(e^x)}{dx} = e^x$$

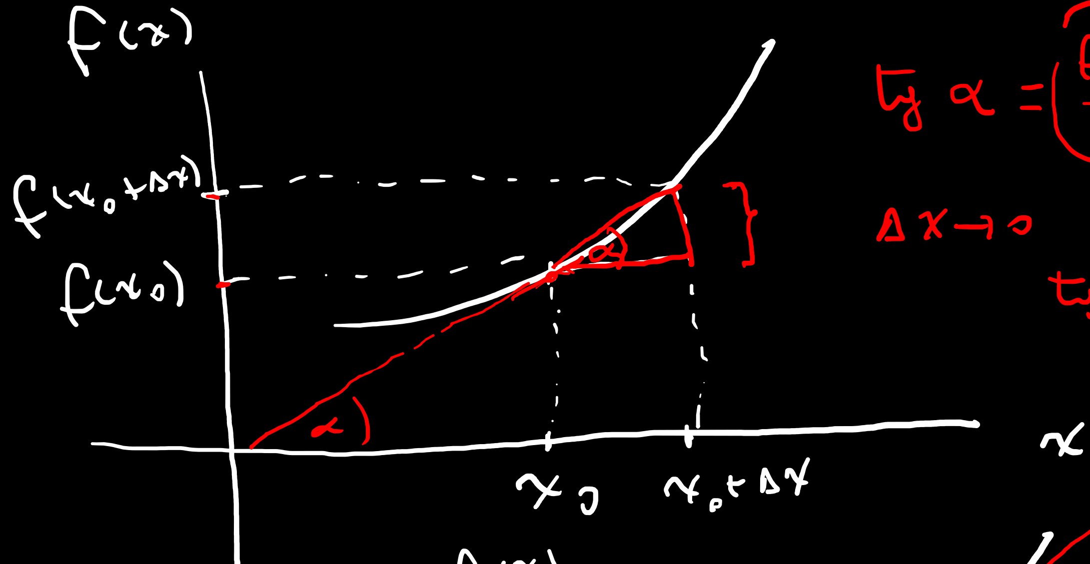
2) Sea $f(x) = e^{3x^2}$

Entonces $f'(x) = e^{3x^2} \cdot 6x$

3) Sea $f(x) = e^{-x^2}$

$$f'(x) = e^{-x^2} \cdot (-2x)$$

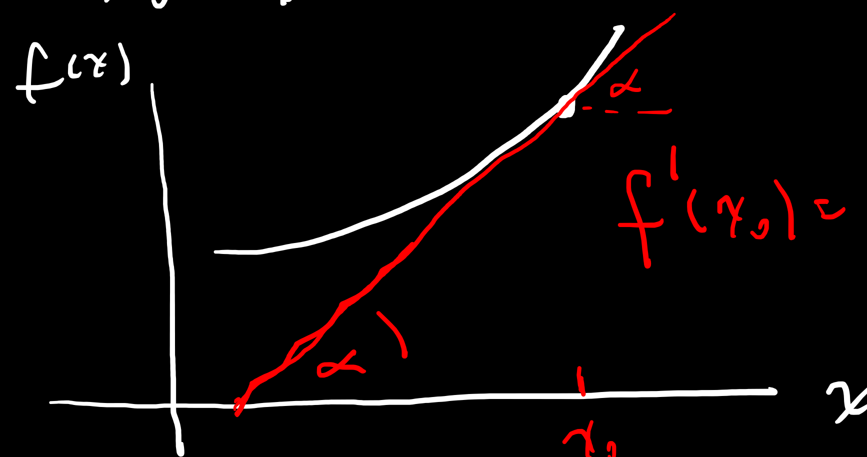
Aplicación de la derivada



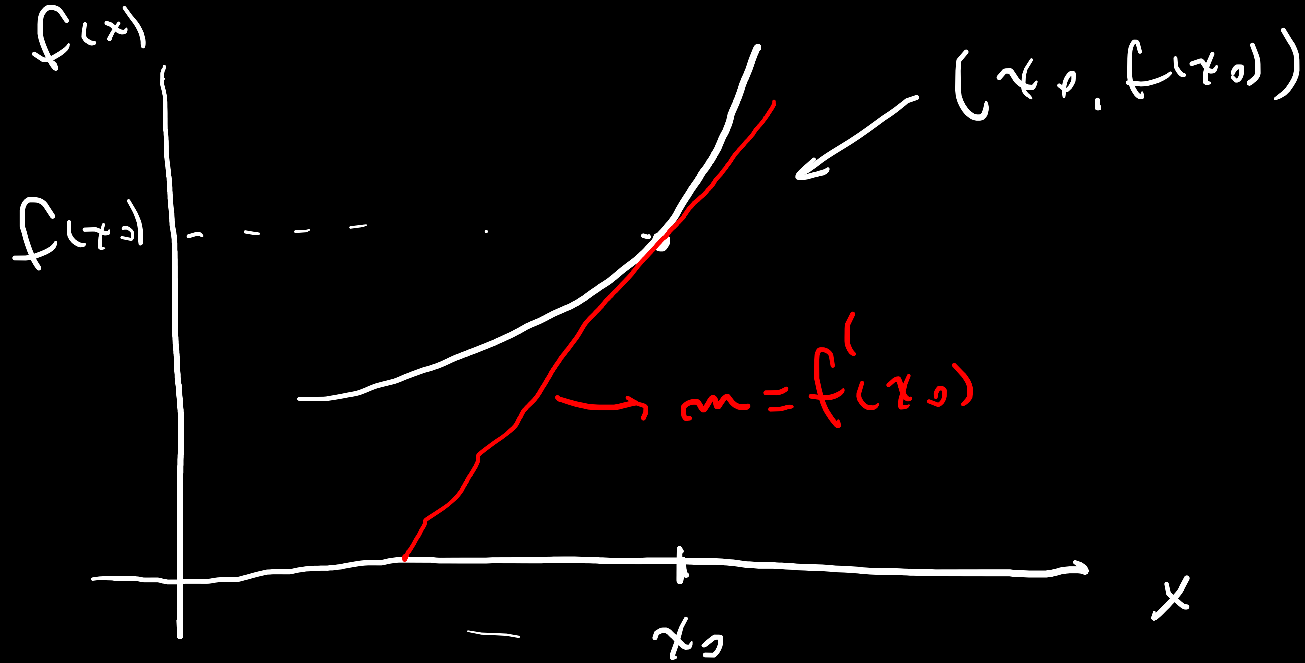
$$\operatorname{tg} \alpha = \frac{f(x_0+\Delta x) - f(x_0)}{\Delta x}$$

$\Delta x \to 0$

$$\operatorname{tg} \alpha = f'(x_0)$$



$$f'(x_0) = \operatorname{tg} \alpha = m$$



¿Cuál es esa recta?

$y = f'(x_0)(x - x_0) + f(x_0)$

La ecuación de la recta es

$$m = \frac{y - f(x_0)}{x - x_0} \rightarrow y = m(x - x_0) + f(x_0)$$

¡Que es la recta!

¿Se acuerdan de la ecuación de una recta?

$$y = mx + b$$

x	y
x_0	y_0
x_1	y_1

$$m = \frac{y_1 - y_0}{x - x_0}$$

$$\boxed{m \; ; \; (x_0, y_0)}$$

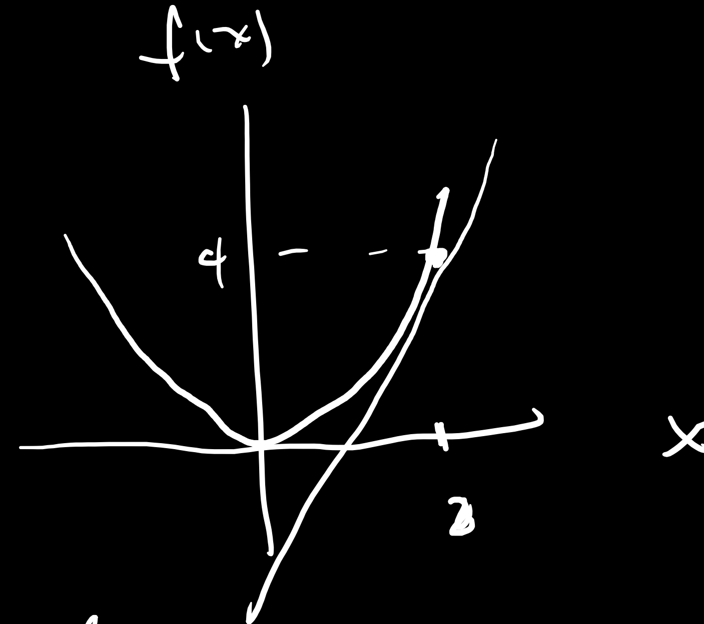
$$m = \frac{y - y_0}{x - x_0} \rightarrow y - y_0 = m(x - x_0)$$

$$\boxed{y = m(x - x_0) + y_0}$$

Ejemplo

$$f(x) = x^2$$

$$x_0 = 2$$



$$f'(x) = 2x$$

$$f'(x_0) = f'(2) = 4$$

$\therefore$ $m = 4$ y el punto es $(2,4)$

$\therefore$ La recta tangente a la curva en el punto $(2,4)$ es

$$y = 4(x-2) + 4$$

$$y = 4x - 8 + 4$$

$$\therefore \boxed{y = 4x - 4}$$

$$f(x) = x^2 \qquad x_0 = 3$$

$$[3, 9]$$

$$f'(x) = 2x \longrightarrow f'(3) = 6 = m$$

$$y = 6(x-3) + 9$$

$$y = 6x - 18 + 9$$

$$\boxed{y = 6x - 9}$$

Encontrar la ecuación de la recta que es tangente a la función $3x^2$, en el punto $x_0 = 1$.

Solución: $f(x) = 3x^2$; $x_0 = 1$

$f'(x) = 6x$; $f'(1) = 6 = m$

y el pto. de la recta es $(1, 3)$

$\therefore$ la recta es $y = 6(x-1) + 3$

$y = 6x - 6 + 3$

$\boxed{y = 6x - 3}$